5.5 The Quadratic Formula

Ex. 1 Find the roots/zeroes/x-int of $y = 2x^2 - 11x + 5$.

let $y = 0$

M 10

A -11

N $\frac{2x}{-10}$ $\frac{2x}{-1}$

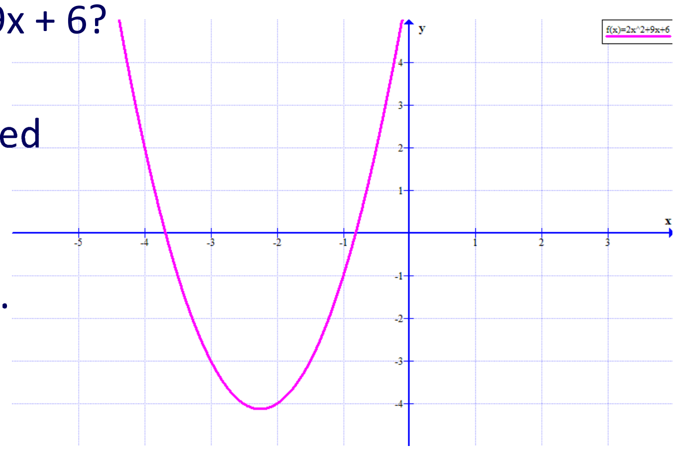
$0 = 2x^2 - 11x + 5$

$= (x-5)(2x-1)$

$x = 5$ $\quad x = \frac{1}{2}$

Can you find the zeroes of $y = 2x^2 + 9x + 6$?

The equation cannot be factored BUT you can see it has two zeroes. So we need another method for finding the zeroes .



Find the zeroes of $y=2(x-1)^2-18$

let $y=0$

$0=2(x-1)^2-18$ 'isolate x'

$18=2(x-1)^2$

$\frac{18}{2}=(x-1)^2$

$9=(x-1)^2$

$\pm\sqrt{9}=\sqrt{(x-1)^2}$

$\pm 3=x-1$

$1\pm 3=x$

$x=1+3$
$=4$

$x=1-3$
$=-2$

$\therefore$ Zeroes at -2 & 4

Solve for x by completing the square $2x^2 - 11x + 5 = 0$

$$2\left(x^2 - \frac{11}{2}x \qquad\right) + 5 = 0$$

$$2\left(x^2 - \frac{11}{2}x + \frac{121}{16} - \frac{121}{16}\right) + 5 = 0$$

$$\tfrac{1}{2}\left(\tfrac{11}{4}\right)^2$$

$$2\left(x^2 - \frac{11}{2}x + \frac{121}{16}\right) - \frac{121}{_8\cancel{16}}\cdot\cancel{2} + 5 = 0$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{121}{8} + \frac{40}{8} = 0$$

$$2\left(x - \frac{11}{4}\right)^2 - \frac{81}{8} = 0$$

Phew!

To derive the Quadratic Formula solve for x if $ax^2 + bx + c = 0$ by completing the square!

$$a\left(x^2 + \frac{b}{a}x \qquad\right) + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = 0$$

$$\tfrac{1}{2}\left(\tfrac{b}{2a}\right)^2$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^{\cancel{2}}}\cdot\cancel{a} + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4ac}{4a} = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 + \frac{(-b^2 + 4ac)}{4a} = 0$$

Now solve

$$2\left(x - \frac{11}{4}\right)^2 = \frac{81}{8}$$

$$\left(x - \frac{11}{4}\right)^2 = \frac{81}{2\cdot 8}$$

$$\left(x - \frac{11}{4}\right)^2 = \frac{81}{16}$$

$$\sqrt{\left(x - \frac{11}{4}\right)^2} = \pm\sqrt{\frac{81}{16}}$$

$$x - \frac{11}{4} = \pm\frac{\sqrt{81}}{\sqrt{16}}$$

$$x = \frac{11}{4} \pm \frac{9}{4}$$

$$x = \frac{11 \pm 9}{4}$$

$$x = \frac{11+9}{4} = 5 \qquad x = \frac{11-9}{4} = \frac{1}{2}$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{a\cdot 4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To find the roots when the equation DOES NOT factor or is not in factored form use :

The Quadratic Formula:

b²-4ac is called the DISCRIMINANT

For $ax^2 + bx + c = 0$, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Time for a song! And another....

Ex. 1 Solve. Give EXACT solutions then decimals.

a) $0 = x^2 - 3x + 1$

$a=1$ $b=-3$ $c=1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

→ Exact Final Answer

$$x = \frac{3+\sqrt{5}}{2} \quad \& \quad x = \frac{3-\sqrt{5}}{2}$$

$x \doteq 2.62$ $x \doteq 0.38$

b) $2x(x - 3) = 7$

$2x^2-6x-7=0$

.... ¨ CANNOT FACTOR!

USE QUAD!

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2-4(2)(-7)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{36+56}}{4}$$

$$x = \frac{6 \pm \sqrt{92}}{4}$$

EXACT

$$x = \frac{6+\sqrt{92}}{4} \quad \& \quad x = \frac{6-\sqrt{92}}{4}$$

$\doteq 3.90$ $x \doteq -0.9$

Ex.2 Solve each of the following using the quadratic formula.

$2x^2 - 5x - 1 = 0$	$x^2 - 30x + 225 = 0$	$3x^2 + 2x + 15 = 0$
		$x = \frac{-2 \pm \sqrt{4 - 4(3)(15)}}{2(3)}$
		$= \frac{-2 \pm \sqrt{-176}}{6}$
		$\therefore$ NO SOL'N
		(it has no intercepts)

Which part of the quadratic formula determines the number of zeros?

To be continued...

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