3.6 Factored Form

Use graphing technology

Investigate:
Graph the equation and note the location of the x-intercepts,
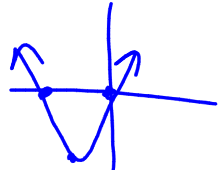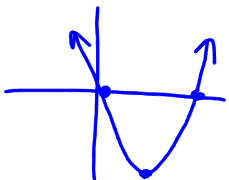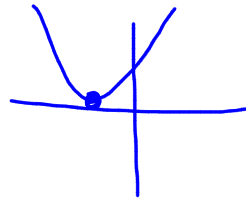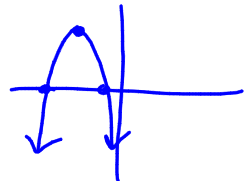axis of symmetry and vertex.

| Equation	x-int.	axis of symmetry	vertex	sketch
$y = (x - 4)(x + 2)$	$x=4$ $(4,0)$ $x=-2$ $(-2,0)$	$\dfrac{4+(-2)}{2}$ $= 1$ $x = 1$	Sub $x=1$ $y=(1-4)(1+2)$ $=(-3)(3)$ $=-9$ $(1,-9)$	
$y = 0.5(x - 5)(x - 1)$	$(5,0)$ $(1,0)$	$x=\dfrac{5+1}{2}$ $x = 3$	Sub $x=3$ $y=0.5(-2)(2)$ $=-2$ $V(3,-2)$	
$y = 2x(x + 4)$	$(-4,0)$ $(0,0)$	$x=-2$	$(-2,-8)$	
$y = (x - 2)(x - 7)$	$(2,0)$ $(7,0)$	$x=\dfrac{9}{2}$	$\left(\dfrac{9}{2},-\dfrac{25}{4}\right)$	
$y = (x + 4)(x + 4)$	$(-4,0)$	$x=-4$	$(-4,0)$	
$y = -3(x + 2)(x + 5)$	$(-2,0)$ $(-5,0)$	$x=-\dfrac{7}{2}$	$\left(-\dfrac{7}{2},\dfrac{27}{4}\right)$	

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Summary: $y = a\,(x - r)\,(x - s)$

same "a" as vertex form

- represents an equation in <u>FACTORED</u> form
- the x-intercepts, or zeros, are r and s
- the axis of symmetry is between the x-intercepts $x = \dfrac{r+s}{2}$

- the x-coordinate of the vertex is the value of the axis of symmetry

- find the y-coordinate of the vertex by substituting the x-coordinate of the vertex in the equation

Ex. 1 Determine the equation of the parabola in factored form. Algebraically determine the value of 'a'.

a)

(1, 8)

(3, 0)

(-1, 0)

b)

(0, 0)

(7, 0)

(1, -3)
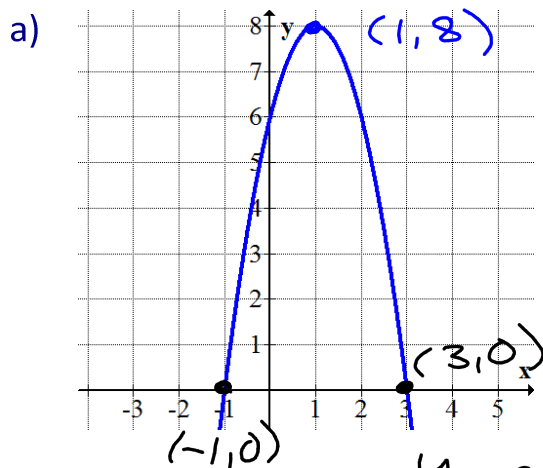
$$y = a(x - r)(x - s)$$

$y = a(x+1)(x-3)$
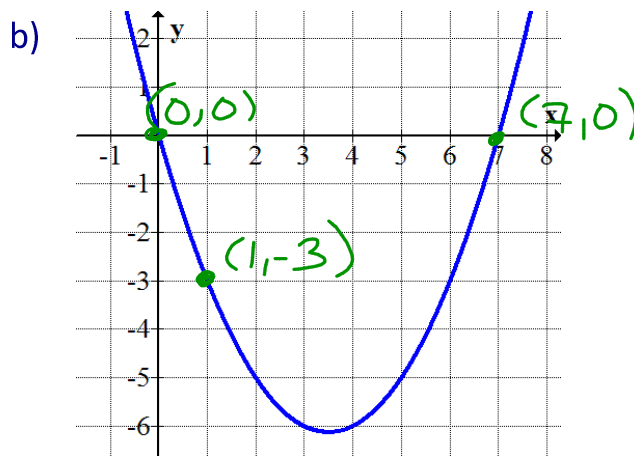
$8 = a(1+1)(1-3)$

$8 = a(2)(-2)$

$8 = a(-4)$

$-2 = a$

$\therefore y = -2(x+1)(x-3)$

$y = a(x-7)(x)$

$-3 = a(1-7)(1)$

$-3 = a(-6)$

$\dfrac{1}{2} = a$

$\therefore y = \dfrac{1}{2}(x-7)(x)$

Ex. 2   Sketch each parabola. Label the x-intercepts and the vertex.

a) $y = (x-3)(x+5)$

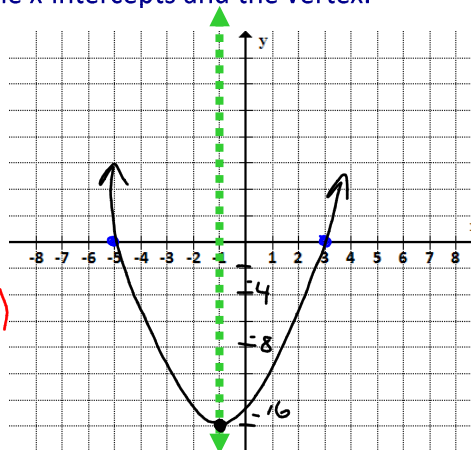$x = 3 \quad \& \quad x = -5$

<u>A.O.S.</u>

$x = \dfrac{-5+3}{2}$

$= -1$

Find y

Sub $x = -1$

$y = (-1-3)(-1+5)$

$= (-4)(4)$

$= -16$

$V(-1, -16)$

b) $y = -0.3(x+2)(x+5)$

$x = -2 \quad \& \quad x = -5$
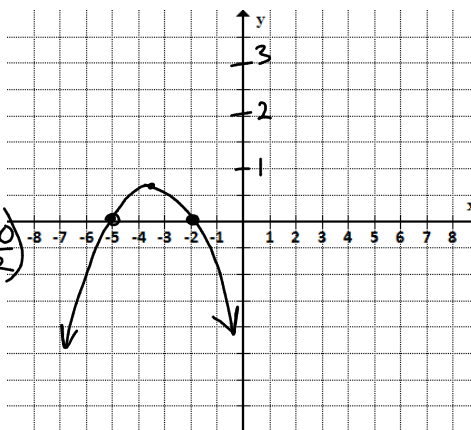
<u>A.O.S.</u>

$x = \dfrac{-2-5}{2}$

$= -\dfrac{7}{2}$

Sub $x = -\dfrac{7}{2}$

$y = -0.3\left(-\dfrac{7}{2} + \dfrac{4}{2}\right)\left(-\dfrac{7}{2} + \dfrac{10}{2}\right)$

$= -0.3\left(-\dfrac{3}{2}\right)\left(\dfrac{3}{2}\right)$

$= 0.675$

$V(-3.5, 0.675)$

c) $y = -\dfrac{1}{2}(x-1)(x+6)$

$x = 1 \quad \& \quad x = -6$

<u>A.O.S.</u>
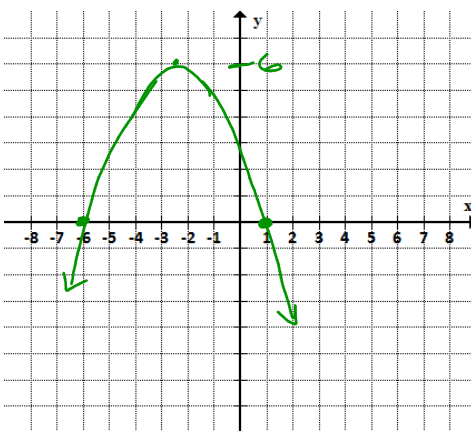
$x = \dfrac{1 + (-6)}{2}$

$= \dfrac{-5}{2}$

Sub $x = -\dfrac{5}{2}$

$y = -\dfrac{1}{2}\left(-\dfrac{5}{2} - 1\right)\left(-\dfrac{5}{2} + 6\right)$

$y = -\dfrac{1}{2}\left(-\dfrac{5}{2} - \dfrac{2}{2}\right)\left(-\dfrac{5}{2} + \dfrac{12}{2}\right)$

$y = -\dfrac{1}{2}\left(-\dfrac{7}{2}\right)\left(\dfrac{7}{2}\right)$
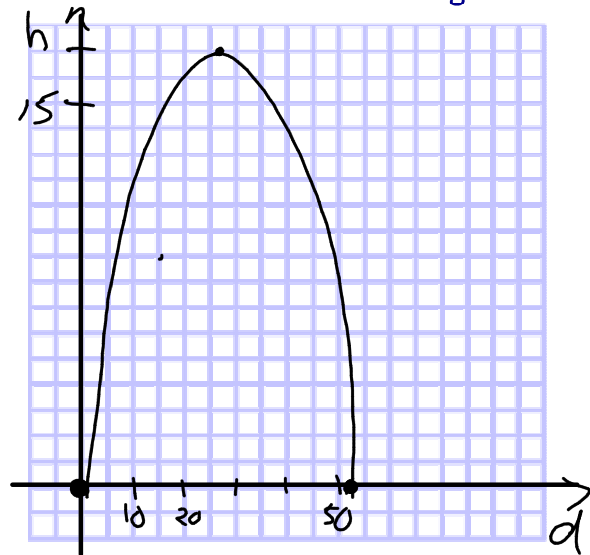
$= \dfrac{49}{8}$

Ex. 3   Chris kicked a ball from the ground. It travelled a horizontal distance of 52 m and reached a maximum height of 17 m.

a)   Draw a sketch of the relation between horizontal distance and height.

A.O.S.

$$x = \frac{0+52}{2}$$

$$= 26$$

$v(26, 17)$

↑ Given

b)   Determine the equation of the relation in factored form.

$$y = a(x-r)(x-s)$$

$$y = a(x-0)(x-52)$$

Sub in $v(26, 17)$

$$17 = a(26-0)(26-52)$$

$$17 = a(26)(-26)$$

$$17 = a(-676)$$

$$-\frac{17}{676} = a$$

$$\therefore y = -\frac{17}{676}(x)(x-52)$$

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Your Turn