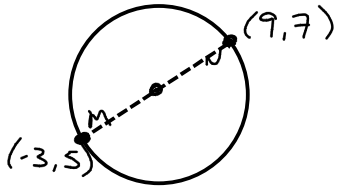
2.5 Problems: Slope, Length and Midpoint

Put it all together now.....

Ex.1 Determine the radius of a circle with endpoints of a diameter M(-3,5) and N(9,7).



Approach 1

① Find midpoint

② Distance from midpoint to either M or N

① $M_{MN} = \left(\frac{-3+9}{2}, \frac{5+7}{2}\right)$

$= (3,6)$

② $\ell_{PN} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$= \sqrt{(3-9)^2 + (6-7)^2}$

$= \sqrt{36+1}$

$= \sqrt{37}$

Approach 2

① Find length MN

② Divide length by 2 for radius

① $\ell_{MN} = \sqrt{(-3-9)^2 + (5-7)^2}$

$= \sqrt{(-12)^2 + (-2)^2}$

$= \sqrt{144+4}$

$= \sqrt{148}$

Diameter

② $r = \frac{d}{2}$

$= \frac{\sqrt{148}}{2}$

$= \frac{\sqrt{4 \times 37}}{2}$

$= \frac{\sqrt{4}\sqrt{37}}{2}$

$= \frac{2\sqrt{37}}{2}$

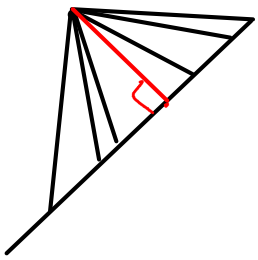
$= \sqrt{37}$

$\frac{\sqrt{148}}{2} \doteq 6.1$

$\therefore$ The radius is $\sqrt{37}$ units

Investigate!

What is the shortest distance from the point to the line?

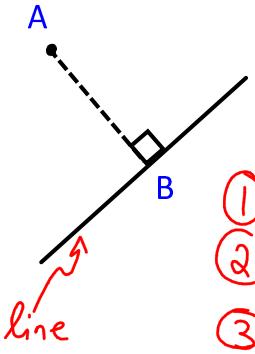


- Draw a line and a point.
- Connect the point and line with several line segments.
- Measure the line segments.
- Which is the shortest? What are its properties?

The one that creates 90°!

The shortest distance from a point to a line is always the length of the segment that is perpendicular to the line.

How do you find this length?



To get the distance from A to B we need...

$$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

① Find slope of our line

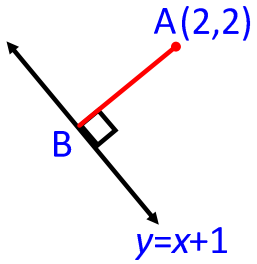
② Slope of AB is negative reciprocal

③ Use $m_{\perp}$ and co-ordinates of A in $y = mx + b$ for line AB

④ Use substitution/elimination to solve the system of eqns to find B.

⑤ Find length AB

Ex. 2 Find the shortest distance from (2,2) to the line $y = x+1$



Find the line AB

① Slope of given line
$y = x+1$
$m = 1$

②
$m_{\perp} = -1$

③ Eqn of AB
Sub: $m = -1$
$(2,2) \Rightarrow y = mx+b$
$2 = -2+b$
$b = 4$
$\therefore y = -x+4$

Find B (intersection)

④ ① $y = -x+4$
② $y = x+1$
① + ② $2y = 5$
$y = \frac{5}{2}$

Sub $y = \frac{5}{2}$ into ②
$\frac{5}{2} = x+1$
$\frac{5}{2} - 1 = x$
$\frac{3}{2} = x$

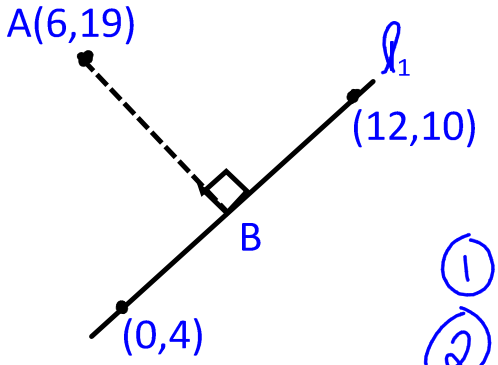
$\therefore B\left(\frac{3}{2}, \frac{5}{2}\right)$

Find length AB

⑤ $l_{AB} = \sqrt{\left(\frac{3}{2}-2\right)^2 + \left(\frac{5}{2}-2\right)^2}$
$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$
$= \sqrt{\frac{1}{4}+\frac{1}{4}}$
$= \sqrt{\frac{1}{2}}$
$= \frac{\sqrt{1}}{\sqrt{2}}$
$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{2}}{2}$

$\therefore$ Distance from A to the line is $\sqrt{\frac{1}{2}}$ units

Ex.3 Given the line containing the point (0,4) and (12,10), determine the distance from A(6,19) to the line.



How is this question different from the last one?

We need to FIND ℓ_1 before hand.....
(No eqn of the line)

① Find eqn of ℓ_1

② Find eqn of AB
- use $m_\perp$ of slope from ℓ_1
- use point A

③ Find intersection of AB & ℓ_1

④ Find length AB
- Using solution of ③ & A

$\ell_{AB} \doteq \sqrt{\frac{576}{5}}$

What we are doing is called analytic geometry!

FBUHL

Basic: Pg. 96 #1de,2bc,6
Regular: Pg. 97 #4d,8,11bcd
Challenge: Pg. 98 #15,17