2.1 Midpoint and Review of y = mx + b

Remember... To write the equation of a line you need 2 points.

- *Parallel* lines have the same slope.

- *Perpendicular* lines have slopes that are negative reciprocals.

- Given two points, find slope using

$$m = \frac{\Delta y}{\Delta x}$$

- *Same x-int* means find the *x*-int by substituting $y = 0$, then use this point, $(x,0)$, as a point on the line.

- Can use any point on the line to substitute along with m to find equation.

Point-Slope Form of a Line: $y = a(x - h) + k$

Determine the equation of the line with slope 2 that goes through the point (3,5).

Your way:

$$y = mx + b$$
$$y = 2x + b$$
$$5 = (2)(3) + b$$
$$5 = 6 + b$$
$$-1 = b$$
$$y = 2x - 1$$

Sub

m x y

My way:

$$y = a(x - h) + k$$
$$y = 2(x - h) + k \quad \text{Substitute the slope for } a$$
$$y = 2(x - 3) + 5 \quad \text{Substitute the point for } (h,k)$$
$$y = 2x - 6 + 5$$
$$y = 2x - 1$$

My Way 2

$$\frac{y_1 - y_2}{x_1 - x_2} = m$$

$$\frac{y - 5}{x - 3} = 2$$

$$y - 5 = 2(x - 3)$$
$$y = 2x - 6 + 5$$
$$y = 2x - 1$$

Examples: Find the equations of the following lines:

a) passes through C(3,-4) and D(-1,7)

x_1, y_1 x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7-(-4)}{-1-3}$$
$$= \frac{11}{-4}$$
$$= -\frac{11}{4}$$

$$y = mx + b$$
$$-4 = -\frac{11}{4}(3) + b$$
$$-4 = -\frac{33}{4} + b$$
$$-4 + \frac{33}{4} = b$$
$$-\frac{16}{4} + \frac{33}{4} = b$$
$$\frac{17}{4} = b$$

$$\therefore y = -\frac{11}{4}x + \frac{17}{4}$$

b) perpendicular to $4x + 3y - 7 = 0$ with the same x-intercept as $2x + 3y - 12 = 0$

$$4x + 3y - 7 = 0$$
$$3y = -4x + 7$$
$$y = -\frac{4}{3}x + \frac{7}{3}$$
$$m = -\frac{4}{3}$$
$$\Downarrow$$
$$m_\perp = \frac{3}{4}$$

x-int

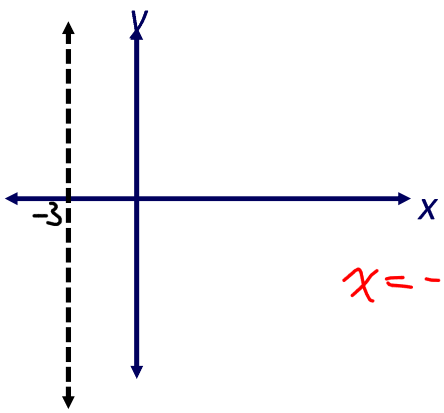
sub $y=0$

$$2x + 3y - 12 = 0$$
$$2x + 3(0) - 12 = 0$$
$$2x = 12$$
$$x = 6$$
$$(6, 0)$$

$$\frac{y - y_1}{x - x_1} = m$$
$$\frac{y - 0}{x - 6} = \frac{3}{4}$$
$$y = \frac{3}{4}(x - 6)$$
$$y = \frac{3}{4}x - \frac{3}{\cancel{4}_2}\left(\frac{\cancel{6}^3}{1}\right)$$
$$y = \frac{3}{4}x - \frac{9}{2}$$

SPECIAL CASES: Horizontal & Vertical Lines

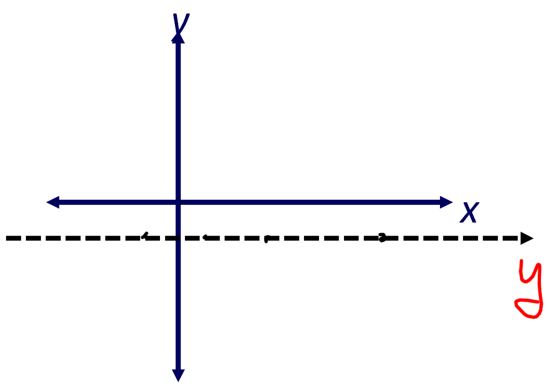
c) a vertical line passing through (-3,5)



the x-coordinate is always -3

$x = -3$

d) a horizontal line passing through (7,-2)



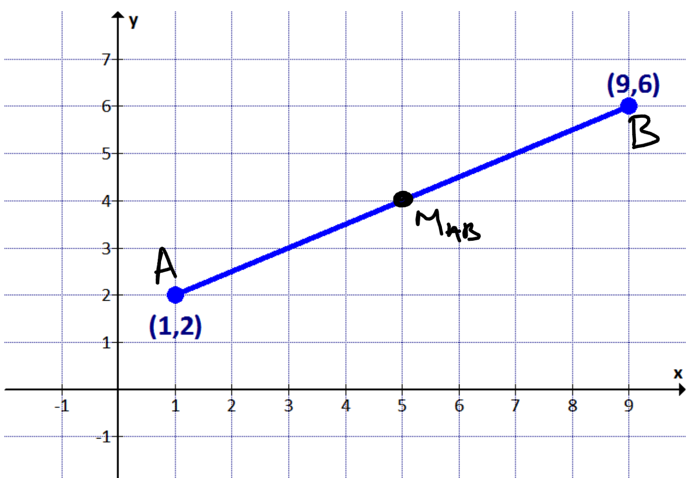
the y-coordinate is always -2

$y = -2$

The Midpoint

Notation: $M(x_M, y_M)$ is used for midpoint.
Remember that m denotes slope!

What are the coordinates of the midpoint of segment AB?

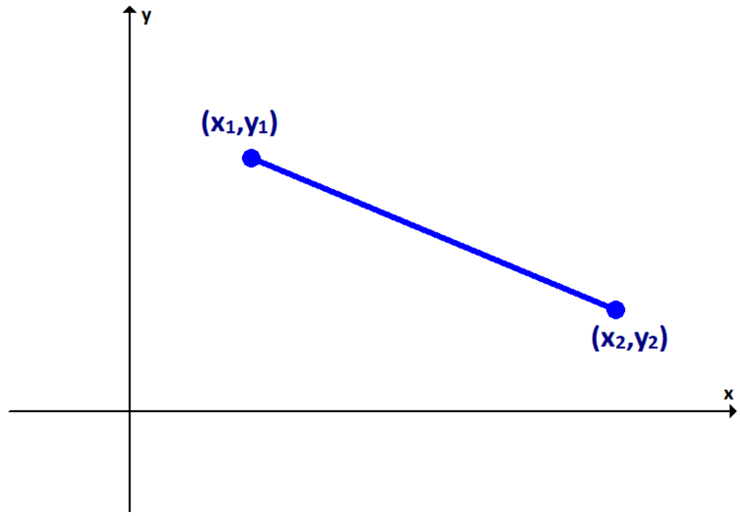


How can you determine the midpoint algebraically given the coordinates of the endpoint?

$$M_{AB} = \left(\frac{1+9}{2}, \frac{2+6}{2}\right)$$
$$= (5, 4)$$

The coordinates of the midpoint of a line segment are found by taking averages:

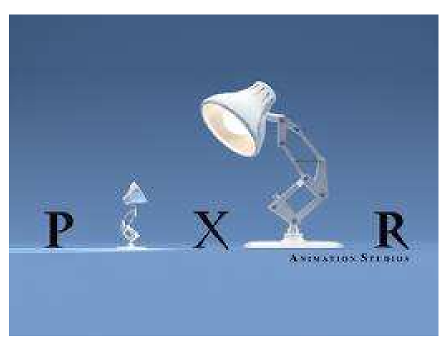
$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$



Ex. 1 Find the midpoint of the line segment AB where A(2,-4) and B(-3,5).

$$M_{AB}=\left(\frac{-3+2}{2},\frac{5+(-4)}{2}\right)$$

$$=\left(-\frac{1}{2},\frac{1}{2}\right)$$



Ex. 2 C(4, -3) is the midpoint of a line segment with endpoints A(7, 5) and B. Determine the coordinates of B.

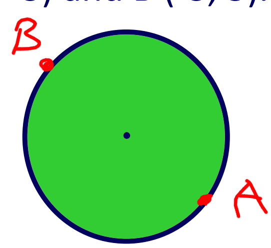
$$(x_M, y_M) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$x_M = \frac{x_1+x_2}{2}$$
$$4 = \frac{7+x_2}{2}$$
$$8 = 7+x_2$$
$$1 = x_2$$

$$y_M = \frac{y_1+y_2}{2}$$
$$-3 = \frac{5+y_2}{2}$$
$$-6 = 5+y_2$$
$$y_2 = -11$$

$\therefore B(1, -11)$

Ex.3 The diameter of a circle has endpoints A(4, -3) and B (-3, 5). Find the centre of the circle.



$$M_{AB} = \left(\frac{4+(-3)}{2}, \frac{-3+5}{2}\right)$$
$$= \left(\frac{1}{2}, 1\right)$$

$\therefore$ Centre of the circle is $\left(\frac{1}{2}, 1\right)$

FBUHL

Basic Pg 66 #1a,2ad,6

Regular Pg 55 #5c,6b,8bc

Pg 66 #3c,4b,12,13a

Challenge Pg 67 #15,16