

## 1.5 Types of Intersections & Equivalent Systems

### A: Types of Intersections



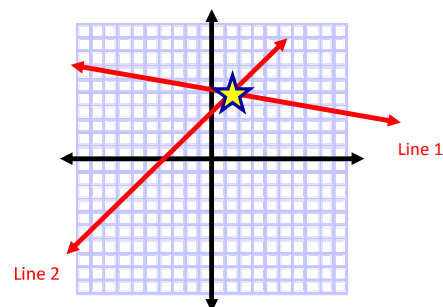
How many solutions can a system of linear equations have?

How can you tell just by looking at the equations?

A linear system can have:

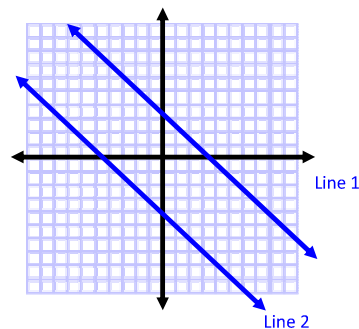
#### ONE SOLUTION

- > The lines intersect at one point  
The two linear equations have
- different slopes
  - y-intercept does not matter



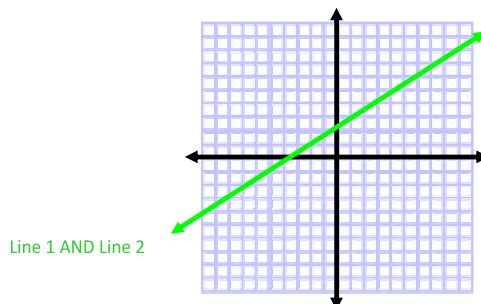
#### NO SOLUTION

- > The lines do not intersect  
The two linear equations have
- same slopes
  - different y-intercepts



#### INFINITE NUMBER OF SOLUTIONS

- > The lines intersect at every point  
The two linear equations have
- same slopes
  - same y-intercepts



Ex. 1 Complete the table.

System #	Equations	Slope	y-int	# of intersections	Solution
1	$y = \frac{2}{3}x - 1$	$\frac{2}{3}$	-1	0	NONE
	$y = \frac{2}{3}x + 2$	$\frac{2}{3}$	2		
2	$y = -\frac{1}{2}x - 1$	$-\frac{1}{2}$	-1	INF	INF
	$x + 2y = -2$ <del><math>y = -\frac{1}{2}x - 1</math></del>	$-\frac{1}{2}$	-1		
3	$y = -\frac{3}{4}x + 1$	$-\frac{3}{4}$	1		(0, 1)
	$y = 2x + 1$	2	1		

What kind of equations give an INFINITE number of solutions?

Equivalent equations

- Same slope & y-int
- Multiples of a given

ie.  $y = 3x - 4$   
 $3y = 9x - 12$   
 $-2y = -6x + 8$

Ex. 2

For what value of  $p$  will the system of linear equations have NO solution?

$$\begin{aligned} \text{a) } x - 2y &= 4 \quad \textcircled{1} \\ y &= px + 1 \quad \textcircled{2} \end{aligned}$$

SAME SLOPE  
DIFF Y-int.

$$\begin{aligned} \text{b) } 3x + y &= 1 \quad \textcircled{1} \\ x + y &= p(x + 2) \quad \textcircled{2} \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad x - 2y &= 4 \\ -2y &= -x + 4 \\ y &= \frac{1}{2}x - 2 \end{aligned}$$

$$\begin{aligned} m &= \frac{1}{2} \\ y &= -2 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad y &= px + 1 \\ p &= m \\ &= \frac{1}{2} \\ \therefore p &= \frac{1}{2} \end{aligned}$$

$$\textcircled{1} \quad y = -3x + 1 \quad \begin{matrix} m = -3 \\ y\text{-int} = 1 \end{matrix}$$

$$\begin{aligned} \textcircled{2} \quad x + y &= px + 2p \\ y &= px - x + 2p \\ y &= (p-1)x + 2p \end{aligned}$$

$$\begin{aligned} p-1 &= -3 \\ p &= -2 \end{aligned}$$

$$\begin{aligned} y\text{-int?} \\ 2p &\Rightarrow 2(-2) \\ &= -4 \text{ Good!} \end{aligned}$$

## B: Equivalent Systems...



Yesterday, I needed 3 Timmy's coffees and 2 donuts to get the day started. That cost me \$8.75. Today, I only needed one coffee and two donuts which cost \$4.25. What is the cost of a coffee and a donut?

Let  $c$  represent cost of a coffee  
 Let  $d$  represent " " " doughnut

$$\textcircled{1} \quad 3c + 2d = 8.75$$

$$\textcircled{2} \quad c + 2d = 4.25$$

$$\textcircled{2} \quad c = 4.25 - 2d$$

Sub into  $\textcircled{1}$

$$3(4.25 - 2d) + 2d = 8.75$$

$$12.75 - 6d + 2d = 8.75$$

$$-4d = -4$$

$$d = 1$$

→ Sub into  $\textcircled{2}$

$$c + 2(1) = 4.25$$

$$c = 4.25 - 2$$

$$= 2.25$$

∴ The cost :

Coffee \$2.25

Doughnut \$1.00

Two Tim's lattes and two cookies cost \$7.10. Five lattes and four cookies cost \$16.95. Determine the cost of a latte and a cookie.

... finish for homework

Answer  
 latte = 2.75  
 cookie = 0.80